

THE NUCLEON SPIN PROBLEM

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Lectures, presented at ITEP Winter School, March 1994

1. Introduction

For many years there exists a permanent non passing away interest to the nucleon spin problem: how the nucleon spin is distributed among its constituents – quarks and gluons. This interest arose essentially after appearance of the results of famous the EMC experiment ¹ on deep inelastic scattering (DIS) of longitudinal polarized muons on longitudinal polarized protons. EMC, using also the earlier data of SLAC ², came to the surprising and contraintuitive conclusion that quarks are carrying a small part of the proton spin projection in the polarized proton. This statement results in excitement of one part of particle physics community and to disappointment of the other. Many ideas, how to overcome this problem were suggested, sometimes new and interesting, sometimes, however, misleading. As a result of these investigations we understand now much more not only about proton spin structure, but also about some others connected with its issues, and this understanding will be with us forever.

In my talk I will try to lay stress on this new understanding, which arised in the last years, taking in mind the aphorism: “The result of calculation is not the number, but the understanding ”.

2. General Relations *

The polarized deep inelastic $e(\mu)$ -nucleon scattering cross section is determined by imaginary part of the forward virtual photon-nucleon scattering amplitude $T_{\mu\nu}^a(p, q)$, antisymmetric in μ, ν . The general form of $ImT_{\mu\nu}^a(p, q)$, following from gauge, P and T invariance is

$$ImT_{\mu\nu}^a(p, q) = \frac{2\pi}{m} \varepsilon_{\mu\nu\lambda\sigma} q_\lambda \left[s_\sigma \left(G_1 + \frac{\nu}{m^2} G_2 \right) - (sq) \frac{p_\sigma}{m^2} G_2 \right], \quad (1)$$

where p and q are the nucleon and virtual photon momenta, s and m – are the nucleon spin and mass. The structure functions G_1 and G_2 depend on two invariants, $q^2 = -Q^2 < 0$ and $\nu = pq$. When Q^2 and ν are large enough, but their ratio $x = Q^2/2\nu$ is fixed, $0 < x < 1$ (so called scaling limit), the scaling relations take place:

$$\begin{aligned} (\nu/m^2) G_1(\nu, q^2) &= g_1(x, Q^2) \\ (\nu/m^2)^2 G_2(\nu, q^2) &= g_2(x, Q^2) \end{aligned} \quad (2)$$

In the parton model g_1 and g_2 are functions of only scaling variable x , in QCD a smooth (logarithmic) dependence of g_1, g_2 on Q^2 appears. In the parton model $g_1(x)$ can be represented through quark distributions in the longitudinally — along the beam polarized nucleon

$$g_1(x) = \sum_{i=u,d,s,\dots} e_i^2 [q_{i+}(x) - q_{i-}(x)], \quad (3)$$

* For more details see ³.

where $q_{i+}(x)$ and $q_{i-}(x)$ are distribution of quarks in the nucleon, polarized along or opposite to the nucleon spin, e_i – are the quark charges. In the parton model x has the meaning of the part of the nucleon momentum, carried by quark in the infinite momentum frame (or for fast moving nucleon, e.g. in the virtual photon-nucleon c.m.s. at $Q^2 \rightarrow \infty$). Therefore, the integrals

$$\Delta q_i = \int_0^1 dx [q_{i+}(x) - q_{i-}(x)] \quad (4)$$

have simple physical meanings: they are equal to the parts of nucleon spin projection, in longitudinally polarized nucleon carried by the quarks of flavour $i = u, d, s, \dots$.

The structure functions G_1, G_2 can be expressed in terms of the virtual photon absorption cross section $\sigma_{3/2}$ and $\sigma_{1/2}$ as well as the quantity σ_I describing the transition from transverse to longitudinal virtual photon polarization (on vice versa) in the forward scattering amplitude. The cross sections $\sigma_{3/2}$ and $\sigma_{1/2}$ correspond to the projections 3/2 and 1/2 of the total photon nucleon spin upon the photon momentum direction.

$$\begin{aligned} \sigma_{1/2} - \sigma_{3/2} &= \frac{8\pi^2\alpha}{\sqrt{\nu^2 - m^2}q^2} \frac{1}{m^2} (\nu G_1 + q^2 G_2) \\ \sigma_I &= \frac{4\pi^2\alpha}{\sqrt{\nu^2 - m^2}q^2} \frac{Q}{m} (G_1 + \frac{\nu}{m^2} G_2) \end{aligned} \quad (5)$$

In turn, $\sigma_{3/2}$, $\sigma_{1/2}$ and σ_I are proportional to the s -channel helicity amplitudes $Im T_{1,-1/2;1,-1/2}$, $Im T_{1,1/2;1,1/2}$ and $Im T_{1,1/2;0,-1/2}$, where the first two indices correspond to final photon and nucleon helicities, and the second two to initial ones. The interference cross section σ_I satisfies the inequality

$$|\sigma_I| < \sqrt{\sigma_T \sigma_0} = \sqrt{R} \sigma_0, \quad (6)$$

where σ_T and σ_0 are the absorption cross sections of transverse and longitudinal virtual photons in nonpolarized scattering, $\sigma_T = (\sigma_{3/2} + \sigma_{1/2})/2$, $R = \sigma_0/\sigma_T$.

The calculations of the structure functions in QCD are performed in the framework of the operator product expansion (OPE). The operators are classified according to twist = dimension - spin. The expansion in twists is equivalent to expansion in inverse powers of Q^2 at fixed x . This expansion starts with twist 2 terms in the case of G_1 , and with twist 3 terms in the case of $G_2(x, Q^2)$, corresponding of scaling behavior of $g_1(x, Q^2), g_2(x, Q^2)$. In perturbative QCD only the evolution with Q^2 of the structure functions can be determined, the values of the structure functions at some fixed $Q^2 = Q_0^2$ are taken from the experiment and used as an input for evolution equations.

The goal of all performed till now experiments was to measure the structure function $g_1(x, Q^2)$. This goal was achieved in the following way. The experimentally measurable quantity is the asymmetry

$$A = \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow} + \sigma^{\uparrow\uparrow}} \quad (7)$$

where $\sigma^{\downarrow\uparrow}, \sigma^{\uparrow\uparrow}$ correspond to the cases, when the spins of longitudinal polarized $\mu(e)$ and nucleon are antiparallel or parallel. The asymmetry A is related to

$$A_1 = (\sigma_{1/2} - \sigma_{3/2})/(\sigma_{1/2} + \sigma_{3/2}) , \quad A_2 = \sigma_I/\sigma_T \quad (8)$$

by

$$A = D(A_1 + \eta A_2) \quad (9)$$

where

$$D = \frac{y(2-y)}{y^2 + 2(1-y)(1+R)} , \quad \eta = \frac{Q}{E} \frac{2(1-y)}{y(2-y)} \quad (10)$$

$y = E - E'$, E and E' are in initial and final $\mu(e)$ energies. It can be shown using the inequality (6), that the second term in (9) is small in practically interesting cases. Then the measurable quantity is proportional to A_1 and the latter determines $g_1(x, Q^2)$ through the relation

$$g_1(x, Q^2) = \frac{F_2(x, Q^2)A_1(x, Q^2)}{2x(1+R)} , \quad (11)$$

where $F_2(x, Q^2)$ is the structure function for nonpolarized DIS. Before going to new developments in the theory of polarized structure functions, I recall some facts which should be well known. However, sometimes a misunderstanding occurs even here.

3. Small x Domain – Regge Behavior

Consider the spin-dependent virtual photon-nucleon forward scattering amplitude

$$T_{\mu\nu}^a(p, q) = \frac{2\pi}{\pi} \varepsilon_{\mu\nu\lambda\sigma} q_\lambda \left\{ s_\sigma S_1(\nu, q^2) + \frac{1}{m^2} [\nu s_\sigma - (sq)p_\sigma] S_2(\nu, q^2) \right\} , \quad (12)$$

$$G_1(\nu, q^2) = \text{Im } S_{1,2}(\nu, q^2) . \quad (13)$$

The functions $S_{1,2}(\nu, q^2)$ satisfy the crossing-symmetry relations

$$S_1(\nu, q^2) = S_1(-\nu, q^2) , \quad S_2(\nu, q^2) = -S_2(-\nu, q^2) . \quad (14)$$

It can be shown, that at $\nu \rightarrow \infty$ and $Q^2 = \text{Const}$ (or $x \rightarrow 0, Q^2 = \text{Const}$) S_1 is contributed by Regge poles, satisfying the condition

$$G(-1)^T \sigma = -1 , \quad (15)$$

where G is G -parity, T - is the isospin and σ is the signature. Then in accord with (12), (14) $\sigma = -1$ and the lowest Regge trajectory contributing to S_1 are $a_1(A_1)$ and $f_1(D)$ meson trajectories

$$S_1(\nu, q^2) \approx \beta_{a_1}(q^2) \nu^{\alpha_{a_1}-1} , \quad g_1(x) \sim x^{-\alpha_{a_1}} . \quad (16)$$

The intercept α_{a_1} is known not quite well, certainly it is negative and about $\alpha_{a_1} \approx -(0.0 - 0.3)$. The same intercept is expected for f_1 trajectory.

Up to the small contribution $\lesssim \nu^{-1.5}$, arising from S_1 (see eq.(5)), the function S_2 is proportional to the helicity amplitude $T_{1,1/2;0,-1/2}$. The Regge poles do not contribute to the latter. This statement directly follows from the factorization theorem for Regge poles. Indeed, the interaction vertex of virtual γ with any Regge pole in this case is a scalar, constructed from vectors $\mathbf{q}, \mathbf{e}_L, \mathbf{e}_T$, linear in the last two, what is impossible. For this reason $S_2(\nu, q^2)$ at large ν is determined by branch points contributions (three pomeron cut etc.)

$$S_2(\nu, q^2) \approx \beta_c(Q^2)/\ln^5 \nu + \sum_{i=P', A_2} \beta_i(Q^2) \nu^{\alpha_i-1} / \ln \nu + \dots \quad (17)$$

4. The Parton Model

As was already mentioned, the spin dependent structure function $g_1(x)$ is represented in the parton model in terms of quark distributions by eq.3 in comparison with the representation of nonpolarized structure function

$$F_2(x) = x \sum_i e_i^2 [q_{i+}(x) + q_{i-}(x)] \quad (18)$$

The standard parametrization of quark distributions at some normalization point $Q^2 = Q_0^2$ (usually $Q_0^2 \sim 5 \text{ GeV}^2$) used in the description on the nonpolazed DIS data is obtained by matching the behavior at small x (Regge domain) with the behavior at large x , $1 - x \ll 1$, following from the quark counting rules:

$$q(x) \equiv q_+(x) + q_-(x) = Ax^{-\alpha}(1-x)^\beta \quad (19)$$

It is expected that for valence quarks $\alpha \approx 0.5$ (ρ - intercept), $\beta \approx 3.0$, for sea quarks $\alpha \approx 1.0$ (experimentally $\alpha \approx 1.2$ is more preferable), $\beta \approx 5$.

A more refined parametrization, which accounts for the fact that $q_+(x)$ and $q_-(x)$ behave differently at $x \rightarrow 1$ $q_-(x)/q_+(x) \sim (1-x)^2$ was suggested by Brodsky (see the review ⁴).

The same parametrization (19) can be applied to spin dependent quark distributions

$$q_+(x) - q_-(x) = Bx^{-\alpha'}(1-x)^{\beta'} \quad (20)$$

where $\alpha' = \alpha_{a_1} \approx -(0.0 - 0.3)$, $\beta' \approx 3$ for valence and $\beta' \approx 5$ for sea quarks.

The parametrizations (19), (20) results in an interesting inequality in the case of strange (sea) quarks ^{5,6}. For strange quarks (19), (20) reduce to

$$\begin{aligned} xs(x) &\equiv x[s_+(x) + s_-(x)] \approx A_s(1-x)^{\beta_s} \quad , \\ s_+(x) - s_-(x) &\approx B_s(1-x)^{\beta_s} \quad , \end{aligned} \quad (21)$$

where it was accepted for simplicity that $\alpha = 1.0, \alpha' = 0$. Since $s_{\pm}(x) \geq 0$, we have an inequality

$$A_s \geq |B_s|, \quad (22)$$

which results in

$$V_{2,s} \equiv \int_0^1 xs(x)dx \geq \int_0^1 dx[s_+(x) - s_-(x)] \equiv \Delta s \quad (23)$$

in the notation (4). The inequality (23) states, that, if the parametrization (23) is correct at some Q_0^2 , then at this Q_0^2 the part of proton spin projection carried by strange quarks cannot exceed the part of proton momentum carried by them. It is clear that small deviation from $\alpha = 1.0, \alpha' = 0.0$, say $\Delta\alpha_T = -0.2, \Delta\alpha' = 0.3$ cannot seriously violate the inequality (23).

5. Sum Rules in the Parton Model

The sum rules for spin dependent structure functions of deep inelastic scattering follows from space-time representation of amplitudes in terms of current commutators:

$$Im T_{\mu\nu}^a(x) = \frac{1}{4} \langle p, s | [j_{\mu}(x), j_{\nu}(0)] | p, s \rangle_{antisym}. \quad (24)$$

Current commutators satisfy the causality condition

$$[j_{\mu}(x), j_{\nu}(0)] = 0 \quad \text{at} \quad x^2 < 0. \quad (25)$$

It can be shown that the domain near light cone corresponds to large Q^2 ⁷. The sum rules can be derived by considering eq.(24) in the limit $x^2 \rightarrow 0, x_0 \rightarrow 0$ and using equal time commutation relations for calculation of current commutators, expressed in terms of free quark fields. Accounting for u, d, s quarks we have from consideration of space-like components $\mu = i, \nu = k$ in (24):

$$Im T_{ik}(x)_{x_0 \rightarrow 0} = -\varepsilon_{ikl} \langle p, s | \frac{1}{3} \left[j_{5l}^3(0) + \sqrt{\frac{1}{3}} j_{5l}^8(0) \right] + \frac{2}{9} j_{5l}^0 | p, s \rangle, \quad (26)$$

where j_{5l}^3, j_{5l}^8 and j_{5l}^0 are isovector, octet and singlet (in SU(3) flavour symmetry) axial currents. The matrix elements in the r.h.s. of (26) are proportional to the nucleon coupling constants with axial currents. The isospin symmetry determines the proton matrix element of isovector current

$$\langle p, s | j_{5l}^3(0) | p, s \rangle_p = -2mg_A s_l, \quad (27)$$

where $g_a = 1.25$ is the neutron β -decay axial coupling constant. On the other side, using the Fourier transformation of eq.1, the l.h.s. of (26) can be represented in

terms of the structure functions (for details see ³). In this way the famous Bjorken sum rule ⁸ arises

$$\int_0^1 [g_1^p(x) - g_1^n(x)] dx = \frac{1}{6} g_A . \quad (28)$$

The Ellis-Jaffe sum rule ⁹ for octet current follows if SU(3) flavour symmetry for baryonic octet β -decays is supposed

$$\int_0^1 g_1^8(x) dx = \frac{1}{24} (3F - D) , \quad (29)$$

where F and D are β -decay axial coupling constants in the baryonic octet.

In a similar way, by considering the behaviour of $Im T_{\mu\nu}^a(x)$ (24) near the tip of light cone the Burkhardt-Cottingham ¹⁰ sum rule for the structure function $g_2(x)$ can be derived

$$\int_0^1 g_2(x) dx = 0 . \quad (30)$$

In the derivation (30) no current algebra is used, only the causality condition (25) and the hypothesis of absence of nontractable singularities in the amplitude at the tip of the light cone are imposed.

The sum rule (30) looks like a superconvergent sum rule, as it was originally derived in Ref.¹⁰. This is not the case, however. Indeed, in accord with (17) we can write the subtractionless dispersion relation for $S_2(\nu, q^2)$

$$S_2(\nu, q^2) = 4\nu \int_{Q^2/2}^{\infty} \frac{G_2(\nu', q^2)}{\nu'^2 - \nu^2} d\nu' . \quad (31)$$

If $S_2(\nu, q^2)$ would decrease faster than $1/\nu$ at $\nu \rightarrow \infty$, then from (31) the sum rule would follow

$$\int_{Q^2/2}^{\infty} d\nu' G_2(\nu', q^2) = 0 , \quad (32)$$

which goes into (30) in the limit $Q^2 \rightarrow \infty$. But due to contributions of pomeron, P' and A_2 cuts, $S_2(\nu, q^2)$ decreases slower than $1/\nu$ (see (17)) and (32) is invalid. That means, that there are higher twist contributions to $g_2(x, Q^2)$ strongly increasing at small x in comparison with lowest twist one, of the order $(1/Q^2)(1/x^2) \ln^{-n} x$, $n \geq 5$. (The sum rule (30) validity was confirmed by QCD sum rule calculation in the lowest order of the operator product expansion in p^2 - nucleon virtuality ¹¹.)

Finally, I discuss the Gerasimov, Drell, Hearn (GDH) sum rule ¹². GDH sum rule refers to the scattering of the real photon on polarized nucleon. But, as we will see, it is closely related to the sum rules for DIS.

In the case of real photon only one amplitude $S_1(\nu, 0)$ survives in (12), because the kinematical factor in front of the other - $S_2(\nu, 0)$ is zero. The function $S_1(\nu, 0)$ satisfies the unsubtracted dispersion relation

$$S_1(\nu, 0) = 4 \int_0^\infty \nu' d\nu' \frac{G_1(\nu', 0)}{\nu'^2 - \nu^2} . \quad (33)$$

Let us go to the limit $\nu \rightarrow 0$ in this relation. According to the F.Low theorem, the terms, proportional to zero and first power in the photon frequency in the photon-nucleon scattering amplitude are expressed via the static characteristics of the nucleon. The direct calculation of Feynman graphs gives

$$S_1(0, 0)_{p,n} = -\kappa_{p,n}^2 , \quad (34)$$

where $\kappa_{p,n}$ are the anomalous proton and neutron magnetic moments. From (33), (34) the GDH sum rule follows

$$\int_0^\infty \frac{d\nu}{\nu} G_{1,p,n}(\nu, 0) = -\frac{1}{4} \kappa_{p,n}^2 . \quad (35)$$

The essential point is that at $Q^2 = 0$ and $\nu \rightarrow 0$ the spin dependent forward Compton amplitude is a constant and has no nucleon pole. That means that in the l.h.s. of the sum rule (35) the elastic contribution is absent — only inelastic processes contribute. That is why the GDH sum rule is very nontrivial.

6. QCD Corrections to Sum Rules

In what follows we use the notation

$$\Gamma_{p,n}(Q^2) = \int_0^1 g_{1,p,n}(x, Q^2) dx . \quad (36)$$

In the parton model

$$\Gamma_{p,n} = \frac{1}{12} \left(\pm a_3 + \frac{1}{3} a_8 + \frac{4}{3} \Sigma \right) , \quad (37)$$

where, for 3 flavours in notation of (4)

$$\begin{aligned} a_3 &= \Delta u - \Delta d ; & a_8 &= \Delta u + \Delta d - 2\Delta s , \\ \Sigma &= \Delta u + \Delta d + \Delta s \end{aligned} \quad (38)$$

and Σ has the meaning of part of the nucleon spin projection carried by quarks. The sum rules, presented in Sec.4 in the supposition of SU(3) flavour symmetry in baryonic β -decays result in equations

$$a_3 = g_A , \quad a_8 = 3F - D . \quad (39)$$

In QCD eq.37 is modified by account of α_s -corrections. The first order corrections in α_s were calculated in ¹³⁻¹⁷, the α_s^2 and α_s^3 corrections to the octet part of the sum rule in ¹⁸, the α_s^2 corrections to the singlet part in ¹⁹. With QCD corrections, calculated in \overline{MS} regularization scheme the sum rules takes the form (for 3 flavours)

$$\begin{aligned} \Gamma_{p,n}(Q^2) = & \frac{1}{12} \left\{ \left[1 - \frac{\alpha_s(Q^2)}{\pi} - 3.6 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^3 \right] \times \right. \\ & \times \left(\pm a_3 + \frac{1}{3} a_8 \right) + \frac{4}{3} \left[1 - \frac{1}{3} \frac{\alpha_s(Q^2)}{\pi} - 1.1 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 \right] \Sigma \Big\} - \\ & - \frac{N_f}{18\pi} \alpha_s(Q^2) \Delta g(Q^2) \end{aligned} \quad (40)$$

where $N_f = 3$ is the number of flavours,

$$\Delta g(Q^2) = \int_0^1 dx [g_+(x) - g_-(x)] , \quad (41)$$

$g_{\pm}(x)$ are the distributions of gluons in the longitudinally polarized nucleon with spin projections along or opposite to the nucleon spin. Δg has the meaning of the part of nucleon spin projection carried by gluons. In eq.(40) a_3, a_8 and Σ are Q^2 independent.

A physically new point in eq.40 is the appearance of gluonic contribution to the sum rule. Although the last term in (40) is an α_s correction and naively it would be expected to vanish in the limit $Q^2 \rightarrow \infty$, in fact it does not, because the $\Delta g(Q^2)$ anomalous dimension is equal to -1 , i.e.

$$\Delta g(Q^2)_{Q^2 \rightarrow \infty} \sim \ln Q^2 \quad (42)$$

and

$$\lim_{Q^2 \rightarrow \infty} \alpha_s(Q^2) \Delta g(Q^2) = Const . \quad (43)$$

For this reason the last term in (40) cannot be considered as a correction vanishing at $Q^2 \rightarrow \infty$, like any other $\alpha_s(Q^2)$ corrections (e.g. the $\sim \alpha_s$ terms in the figure bracket in (40), where a_3, a_8 and Σ have zero anomalous dimensions).

On the other side from the nucleon spin projection conservation it follows that

$$\frac{1}{2} \Sigma + \Delta g(Q^2) + L_z(Q^2) = \frac{1}{2} , \quad (44)$$

where L_z is the orbital momenta contribution. Therefore, the logarithmically increasing with Q^2 $\Delta g(Q^2)$ means that $L_z(Q^2) \sim \ln Q^2$ at large Q^2 , is negative and compensates $\Delta g(Q^2)$. As a direct consequence the quark model of nucleon with quarks in S -states cannot work at high Q^2 .

There was a wide discussion in the past years if gluonic contribution $\Delta_g \Gamma_{p,n}$ to $\Gamma_{p,n}$ (the last term in eq.(40)) is uniquely defined theoretically or is not ^{17,20-32}. The problem is that gluonic contribution to the structure functions, described by imaginary part of the forward γ_{virt} - gluon scattering amplitude (Fig.1) is infrared dependent. Since in the infrared domain the gluonic and sea quark distributions

Fig. 1

The photon-gluon scattering diagrams, the wavy, dashed and solid lines correspond to virtual photons, gluons and quarks.

are mixed and their separation depends on the infrared regularization scheme, a suspicion arises that $\Delta_g \Gamma_{p,n}$ can have any value. This suspicion is supported by the fact that in the lowest order in α_s the terms, proportional to $\ln Q^2$ are absent in $\Delta_g \Gamma_{p,n}$ and this contribution looks like next to leading terms in nonpolarized structure functions, where such an uncertainty is well known.

In order to discuss the problem consider the gluonic contribution $g_{1p}(x, Q^2)_{gl}$ to the proton structure function $g_{1p}(x, Q^2)$, described by evolution equation

$$g_{1p}(x, Q^2)_{gl} = N_f \frac{\langle e^2 \rangle}{2} \int_x^1 \frac{dy}{y} A\left(\frac{x}{y}\right) [g_+(y) - g_-(y)] , \quad (45)$$

where the asymmetry $A(x_1)$ is determined by the diagrams of Fig.1. The calculation of the asymmetry $A(x_1)$, $x_1 = -q^2/2pq$ results in the appearance of integrals

$$\int \frac{d^2 k_\perp}{(k_\perp^2 - x_1(1-x_1)p^2 + m_q^2)^n} , \quad n = 1, 2 ,$$

which are infrared dependent. To overcome this problem it is necessary to introduce the infrared cut-off (or infrared regularization), to separate the domain of large k_\perp^2 , where perturbative QCD is reliable, from the domain of small k_\perp^2 . The contribution of the latter must be addressed to noncalculable in perturbative QCD parton distribution. Such a procedure is legitimate because of factorization theorem, which states that the virtual photoabsorption cross section on the hadronic target h , $\sigma_h^\gamma(x, Q^2)$ can be written down in the convolution form

$$\sigma_h^\gamma(x, Q^2) = \sum_i \sigma_i^\gamma(x, Q^2, M^2) \otimes f_{i/h}(x, M^2) , \quad (46)$$

where σ_i^γ is the photoproduction cross section on i^{th} parton ($i = q, \bar{q}, g$), $f_{i/h}$ are the parton distributions in a hadron h , \otimes stands for convolution. Both σ_i^γ and $f_{i/h}$ depend on the infrared cut-off M^2 , but the physical cross section σ_h^γ is cut-off independent. The variation of M^2 corresponds to redistribution among partons: the trade of gluon for sea quarks.

As follows from (45)

$$\Delta_g \Gamma_p = N_f \frac{\langle e^2 \rangle}{2} \Delta_g \overline{A(M^2)}, \quad (47)$$

where

$$\overline{A(M^2)} = \int_0^1 dx_1 A(x_1, M^2). \quad (48)$$

The convenient way is to introduce cut-off in k_\perp^2

$$k_\perp^2 > M^2(x_1, p^2) \quad (49)$$

Generally, M^2 may depend on x_1 and p^2 . For example, the cut-off in quark virtuality in Fig.1 $-k^2 > M_0^2$ corresponds to the form of (49) with $M^2 = (1 - x_1)(M_0^2 + p^2 x_1)$ if $x_1 < -M_0^2/p^2$.

In the calculation of the diagrams of Fig.1 it is reasonable to neglect the light quark masses in comparison with gluon virtuality p^2 , since we expect that $|p^2|$ is of order of characteristic hadronic masses, $|p^2| \sim 1 \text{ GeV}^2$ ²⁰. Then introducing the infrared cut-off (49) we have³¹

$$\overline{A(M^2)} = -\frac{\alpha_s}{2\pi} \left\{ 1 - \int_0^1 dx_1 (1 - 2x_1) [\ln r - r] \right\}, \quad (50)$$

where

$$r = \frac{x_1(1 - x_1)p^2}{x_1(1 - x_1)p - M^2(x_1, p^2)}. \quad (51)$$

If $M^2 = \text{Const}$ - a rectangular cut-off in k_\perp^2 -, the integral in (50) vanishes (the integrand is antisymmetric under substitution $x_1 \rightarrow 1 - x_1$). Then $\bar{A} = -\alpha_s/2\pi$ and we obtain the gluonic contribution to $\Gamma_{p,n}$ (40). However, other forms of $M^2(x_1, p^2)$ result in different values of $\bar{A(M^2)}$, what support the claim²¹ that $\bar{A(M^2)}$ is cut-off dependent. Even more, if we put $p^2 = 0, m_q^2 \neq 0$ - the standard regularization scheme in the calculation of nonpolarized deep inelastic scattering - we will find $\bar{A} = 0$ ²⁹. This result is, however, nonphysical because the compensation of $-\alpha_s/2\pi$ term in \bar{A} arises from soft non-perturbative domain of $k_\perp^2 \sim m_q^2$, which must be attributed to sea quark distribution.

Although generally \bar{A} , as well as Δg and $\Delta_g \Gamma_p$, are infrared cut-off dependent, a special class of preferable cut-off's can be chosen. In OPE the mean asymmetry \bar{A} is proportional to the one-gluon matrix element of axial current

$$\Gamma_{\mu\lambda\sigma}(0, p, p) = \langle g_1 \varepsilon_\lambda | j_{\mu 5}(0) | g_1 \varepsilon_\sigma \rangle, \quad (52)$$

at zero momentum transfer. It can be shown ³¹, that this quantity is proportional to the divergency $l_\mu \Gamma_{\mu\lambda\sigma}(l, p_1, p_2)$, $l = p_2 - p_1$, in the limit $l^2 \rightarrow 0$, i.e. to the one gluon matrix element of axial anomaly. The standard expression for axial anomaly corresponds to $M^2 = Const$ and $\bar{A} = -\alpha_s/2\pi$ (although it was demonstrated ³¹, that the same infrared dependence persists here also). Therefore, the cut-off in $k_\perp^2 < M^2 = Const$ is preferable, since it preserves the standard form of axial anomaly. In this case the axial anomaly can be considered as a local probe of gluon helicity. Some care, however, is necessary, when the calculations with this cut-off are compared with HO calculation in nonpolarized scattering, where, as a rule, another regularization procedure $-p^2 = 0, m_q^2 \neq 0$ - is used.

The gluonic contribution to the spin dependent proton structure function $g_{1p}(x, Q^2)_{gl}$ can be found experimentally by measuring inclusive two jets production in polarized DIS ²⁰. However, only large - k_\perp^2 component of Fig.1 diagrams can be determined in this way, the small - k_\perp^2 component is unmeasurable, since in this case it is impossible to separate one jet from two jets events ³¹. This fact is in complete accord with the formulated above statement about the arbitrariness of infrared cut-off.

What numerical value of gluonic contribution to $\Gamma_{p,n}(Q^2)$ can be expected? Naively, we can say, that since gluons are carrying about 50% of proton momentum $< x >_{gl} \simeq 0.5$ *, they can carry the same amount of proton spin projection, $\Delta g \approx 1/4 - 1/2$ (see (44)). The same conclusion follows from the simplest parametrization of gluon distribution like (19), (20), or from the parametrization, suggested by Brodsky ⁴. If $\Delta g \approx 1/2$ at $Q^2 = 10 \text{ GeV}^2$ ($\alpha_s(10 \text{ GeV}^2) \approx 0.25$ at $\Lambda_{QCD} = 200 \text{ MeV}$) then

$$\Delta_g \Gamma_{p,n} \approx 0.0066 . \quad (53)$$

Earlier Brodsky and Schmidt ³³ basing on the model of bound-state nucleon and counting rules suggested a different parametrization and obtained $\Delta g \approx 1.2$ at $Q^2 \sim 1 \text{ GeV}^2$. Up to terms of order α_s^2 the perturbative evolution of $\Delta g(Q^2)$ with Q^2 is given by ^{23,35}

$$\Delta g(Q^2) = \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \left\{ 1 + \frac{2N_f}{b\pi} [\alpha_s(Q^2) - \alpha_s(\mu^2)] \right\} \Delta g(\mu^2) + \frac{4}{b} \left[\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} - 1 \right] \Sigma(\mu^2) , \quad (54)$$

where $b = 11 - (2/3)N_f = 9$, μ^2 - is the normalization point. As follows from (54), if the value $\Delta g = 0.25$ is accepted at $\mu^2 = 1 \text{ GeV}^2$, then $\Delta g(10 \text{ GeV}^2) \approx 0.6$ (at $\Lambda_{QCD} = 200 \text{ MeV}$ and $\Sigma(\mu^2) = 0.59$ - see Sec.6). In order to have higher values of $\Delta g(10 \text{ GeV}^2)$ it is necessary to start from lower normalization point.

* This number refers to typical Q^2 in DIS experiments, $Q^2 \sim 5 \text{ GeV}^2$. At $Q^2 \gtrsim 2 \text{ GeV}^2 < x >_{gl}$ only slightly depends on Q^2 , but is decreasing at low Q^2 up to $< x >_{gl} \approx 0.3$ at $Q^2 = 0.3 \text{ GeV}^2$ ³⁴. Since $\Delta g(Q^2)$ increase as $\ln Q^2$ at high Q^2 , the conclusion $\Delta g \lesssim 0.5$ is valid if we assume, that gluonic distribution is described by equations like (19), (20) at $Q^2 \sim 2 - 10 \text{ GeV}^2$.

E.g. $\Delta g(10\text{GeV}^2) = 3$ requires $\Delta g = 1$ at $\mu^2 = 0.3\text{GeV}^2$ ³⁵. The large value of Δg in the latter case is achieved due to increasing of $g_+(x) - g_-(x)$ at small x and the problem arises if such increasing is compatible with experiment and does it proceeds in the domain of x , measured in the existing experiments. There are different opinions about this subject^{30,31,35-37}. Perhaps the problem must be reinvestigated in the light of new experimental data.

It is interesting to mention that in the ratio of Gross-Llewellyn-Smith and Bjorken sum rules the perturbative QCD corrections cancel up to α_s^2 ¹⁸ (see also the discussion of this point in⁴). Therefore the experimental examination of this prediction at moderate Q^2 could be also a good check of perturbative QCD and the role of higher twist terms.

Let us turn now to twist four corrections to the sum rules (40). The general theory of these corrections was developed by Shuryak and Vainstein³⁸. According to this theory the twist 4 corrections to (40) are proportional to the one-nucleon matrix elements of the operators

$$\langle N | U_\mu^{S,NS} | N \rangle = \langle N | g\bar{q}\tilde{G}_{\mu\nu}^a\gamma_\nu\frac{1}{2}\lambda^a(1,\tau_3)q | N \rangle, \quad (55)$$

where $\tilde{G}_{\mu\nu}^a$ is the dual gluonic field tensor, $S(NS)$ are singlet (nonsinglet) in flavour operators, $1(\tau_3)$ in the r.h.s. of (55) correspond to $S(NS)$. In (55) q means u and d and the contribution of strange quarks is omitted. An attempt to determine the matrix elements (55) in the framework of QCD sum rule 3-point function calculations^{39,40} was done by Balitsky, Braun, Kolesnichenko⁴¹. Their final results are

$$\Delta_{twist4}\Gamma_{p,n} = -\frac{1}{18}(4,1)\frac{0.09 \pm 0.06}{Q^2}\text{GeV}^2 + \frac{2}{9}\frac{m^2}{Q^2}\int_0^1 x^2 g_{1,p,n}(x)dx. \quad (56)$$

The magnitude of twist 4 correction following from (56) is very small. Even with account of (negative) error it comprises $\sim -1 \cdot 10^{-3}$ at $Q^2 = 10\text{GeV}^2$ for proton as well as for neutron.

Unfortunately, I cannot believe that the result (56) is reliable for the following reasons. In finding the vacuum expectation values induced by external axial field - a quantity, which essentially determines the final answer - the authors of ref.⁴¹ have really taken the octet field instead of the singlet one and used the dominance of massless goldstones (π or η) which is incorrect for the singlet field case. For the nonsinglet case the situation is, in principle better. But, unfortunately, in this case the main contribution, to the sum rule used by the authors, comes from the highest order accounted term in the OPE - the term $\sim m_0^2 \langle \bar{\psi}\psi \rangle^2$ of dimension 8. It is possible that in this case the contribution of unaccounted terms of OPE is essential or even that the OPE series is divergent at characteristic values of the Borel parameter $M^2 = 1\text{GeV}^2$ used in the sum rule. On the physical side of the sum rule the contribution of continuum comprises 80% and the nucleon pole term from which

Γ_{p-n} was obtained gives only 20%. This also spoils the accuracy of the calculations. Finally, the result of ⁴¹ (unlike the other results obtained by the QCD sum rule) depends on the ultraviolet cut-off. This circumstance introduces a noncontrollable uncertainty into the calculation.

7. Nucleon Axial Coupling Constants

As was demonstrated in the previous Sections, according to the sum rules the integrals

$$\Gamma_{p,n} = \int g_{1,p,n}(x)dx$$

are expressed through the nucleon matrix elements of isovector, octet and singlet axial currents, a_3, a_8 and Σ . The magnitude of a_3 , determining Bjorken sum rule, is well known. According to isospin invariance it is equal to the axial coupling constant in neutron β -decay, $a_3 = g_A = 1.257 \pm 0.003$ ⁴². The theoretical accuracy in this number is given by the accuracy of isospin symmetry, i.e. it is of order 1%. In a similar way a_8 is proportional to the matrix element of octet axial current

$$a_8 = \sqrt{3}s_\mu < p, s | j_{\mu 5}^{(8)} | p, s > / 2m . \quad (57)$$

This matrix element can be found if $SU(3)$ flavour symmetry in baryonic octet β -decays is assumed. Then

$$a_8 = 3F - D = 0.59 \pm 0.02 \quad (58)$$

and the numerical value in (58) corresponds to the best fit ⁴³ of neutron and hyperon β -decays in $SU(3)$ symmetry. The use of $SU(3)$ symmetry in determination of a_8 was questioned by Lipkin ⁴⁴. Indeed, the nondiagonal matrix elements enter neutron and hyperon β -decays, while the diagonal one appears in (57). If $SU(3)$ symmetry is badly violated, it is possible that the latter has nothing to do with the formers. The accuracy of $SU(3)$ symmetry in the matrix elements of octet axial current can be estimated in the following way ⁶. In $SU(3)$ symmetry the interesting for us combination $3F - D$ can be found from any pair of β -decays. If $SU(3)$ is strongly violated we would expect that the values, determined from various pairs of β -decays are different. Experiments tell us ⁴²

$$\begin{aligned} (3F - D)_{np, \Lambda p} &= -3g_{A,np} + 6g_{A,\Lambda p} = 0.537 \pm 0.09 , \\ (3F - D)_{np, \Sigma n} &= g_{A,np} + 2g_{A,\Sigma n} = 0.577 \pm 0.034 , \\ (3F - D)_{\Lambda p, \Sigma n} &= (3/2)(g_{A,np} + g_{A,\Sigma n}) = 0.567 \pm 0.034 , \\ (3F - D)_{\Xi \Lambda} &= 3g_{A,\Xi \Lambda} = 0.75 \pm 0.15 . \end{aligned} \quad (59)$$

We see that all values (59) agree very well with (58) and, at least, the value of $3F - D$ less than 0.5 is completely excluded.

The matrix element a_8 (57) was also determined theoretically by QCD sum rule method ⁴⁵. In this calculation no $SU(3)$ flavour symmetry of baryonic β -decays was assumed. The result

$$a_8 = 0.5 \pm 0.2 \quad (60)$$

is in agreement with (58).

To estimate the proton matrix element of singlet axial current (or Σ) is a more complicated, till now nonsolved problem. Naively, by analogy with a part of the proton momentum, carried by quarks, which is about 50% at $Q^2 \sim 5 - 10 \text{ GeV}^2$, we expect, that $\Sigma \approx 0.5$. The same estimate follows also from the assumption that strange sea is nonpolarized (the original Ellis-Jaffe ⁹ assumption). Then $\Sigma \approx a_8$ given by (58).

On the other side, assuming that the Skyrme model is suitable for the description of proton constituents, Brodsky, Ellis and Karliner ⁴⁶ argue that Σ is small of order $1/N_c$ in the limit of large number of colours N_c . Indeed, in the Skyrme model the nucleon is built from octet of goldstones, which is decoupled from singlet axial current at $N_c \rightarrow \infty$. The Skyrme model, probably, gives a good description of nucleon periphery. However, the description with this model of deep inside nucleon structure, which is measured in deep inelastic scattering, is questionable (see also ⁴⁷).

In ref.48 an attempt was performed to calculate the nucleon matrix element of singlet axial current $a_0 = \Sigma$ by QCD sum rules in the similar way as it was done in the calculation of a_8 in ⁴⁵. The main difference with a_8 calculation is that in this case the anomaly plays an important role in the vacuum expectation value induced by external axial field, which essentially determines the final result. This problem was overcome in ⁴⁸, but nevertheless the result was negative: an attempt to determine Σ fails, since it was found that the OPE breaks down for singlet axial current in longitudinal and transverse parts of polarization operators and/or in the NNA vertex function. The physical consequence of this consideration is that one should expect a noticeable Okubo-Zweig-Iizuka (OZI) rule violation in the nonet of axial 1^{++} mesons.

The authors of Refs.24,49 tried to use the so called “ $U(1)$ Goldberger–Treiman” relation

$$F_{\eta_0}(0)g_{\eta_0 NN} = 2ma_0 \quad (61)$$

to determine a_0 . Unfortunately, no definite conclusion could be obtained in this approach without serious additional hypothesis. The reason for this comes from the fact that due to anomaly η_0 in (61) is not a goldstone and not a physical η' meson. Perhaps, the troubles appearing in this approach are of the same origin as in ⁴⁸.

8. The Experimental Data on $g_1(x)$ and Their Interpretation

The “Sturm und Drang” period in the investigation of nucleon spin structure started after appearance of famous EMC results ¹, where the structure function $g_{1p}(x)$ was measured in the interval $0.015 < x < 0.47$ at the mean $\bar{Q}^2 = 10.7 \text{ GeV}^2$. Combining their data with earlier SLAC ² data, where the measurements were made at $0.1 < x < 0.65$ at $\bar{Q}^2 \approx 3 \text{ GeV}^2$ and performing the extrapolation in the domain $x < 0.015$ and $x > 0.65$, EMC obtained

$$\Gamma_p = 0.126 \pm 0.010(\text{statist}) \pm 0.015(\text{system.}) , \quad \bar{Q}^2 = 10.7 \text{ GeV}^2 \quad (62)$$

Compare (62) with the simplest theoretical version — the Ellis–Jaffe sum rule, — eq.40, where the gluonic contribution is neglected and it is supposed that strange sea is nonpolarized, $\Delta s = 0$. Then $\Sigma = a_8$ and using (58), we have at $\alpha_s(Q^2 = 10.7 \text{ GeV}^2) = 0.25$ ($\Lambda_{QCD} = 200 \text{ MeV}$)

$$\Gamma_p^{EJ} = 0.171 \pm 0.004 , \quad (63)$$

where the error comes from the error in (58) and from the uncertainty in the α_s correction, which could be of the order of α_s^3 term (besides of uncertainty in Λ_{QCD}). The Ellis–Jaffe prediction disagrees with (62) by 2.6 standard deviations.

Reject now the hypothesis $\Delta s = 0$, but still neglect gluonic contribution. Using experimental value (62) we can determine from (39), (40), (58) the values $\Delta u, \Delta d, \Delta s$ and Σ .

$$\begin{aligned} \Delta u &= 0.785 \pm 0.06 , & \Delta d &= -0.47 \pm 0.06 , & \Delta s &= -0.14 \pm 0.06 , \\ \Sigma &= 0.17 \pm 0.17 . \end{aligned} \quad (64)$$

The results (64) as originally was stressed in ^{1,46} are very strange and unexpected: the part of proton spin projection carried by quarks is small and compatible with zero, strange quarks are carrying a large amount of proton spin in contradiction with the quark model and inequality (23). Indeed the experimental data on DIS give ⁵⁰

$$V_{2,s} = 0.026 \pm 0.006 \quad (65)$$

and (23) is strongly violated.

The account of gluons does not improve the situation essentially, if $\Delta g \approx 0.5$ at $Q^2 = 10 \text{ GeV}^2$. Using (53), we find that in this case Δs and Σ increase in comparison with (64) correspondingly by 0.02 and 0.07. If, we assume that $\Delta g(10 \text{ GeV}^2) \approx 2$, then

$$\Delta s = -0.06 \pm 0.06 ; \quad \Sigma = 0.42 \pm 0.17 , \quad (66)$$

the inequality (23) is fulfilled within the errors and Σ does not contradict the naive expectations. The value $\Delta g(10 \text{ GeV}^2) = 2$ can be achieved, if $\Delta g(1 \text{ GeV}^2) = 1.2$ what was suggested in ³³. This solution means, however, that the constituent quark

model would be strongly violated at $Q^2 \sim 1 \text{ GeV}^2$, since, according to (44) we would have $L_z(1 \text{ GeV}^2) \approx -1$.

Leaving aside the discussion of the scenario with large Δg , let us consider if it is possible to have large $|\Delta s|$, violating the inequality (23). Qualitatively, the problem can be formulated as follows: could it be, that in one experiment we see a small number of strange quark pairs in the proton and a much larger one in the other? The answer is ⁵¹: since the number of $\bar{s}s$ pairs is not conserved, the question how many strange quark pairs are in the proton is not correctly formulated until one specify the operator, with which the proton strange content is measured. The measurements of a part of proton momentum, carried by s -quarks corresponds to the matrix element of the s -quark energy momentum tensor operator, $V_{2s} \sim \langle p | \theta_{\mu\nu}^s | p \rangle$, the measurement of proton spin projection, carried by s -quark corresponds to the s -quark axial current operator $\Delta s \sim \langle p | j_{\mu 5}^s | p \rangle$. The small value of V_{2s} and a large of Δs would mean, that in the case of the $\theta_{\mu\nu}^s$ operator the transitions $\bar{s}s \rightarrow \bar{u}u + \bar{d}d$ are suppressed, while this is not the case for the $j_{5\mu}^s$ operator — the OZI rule works well in the first case and is violated in the second. This circumstance is not surprising. In the meson nonets the situation is similar: the OZI rule works well in the case of vector and tensor mesons and is 100% violated in pseudoscalar nonet. The latter phenomenon was explained theoretically by attributing $\bar{s}s \rightarrow \bar{u}u + \bar{d}d$ transitions in mesons to nonperturbative QCD effects — the instantons ⁵². In the field of instanton the transitions $\bar{s}s \rightarrow \bar{u}u + \bar{d}d$ proceed in pseudoscalar and longitudinal axial channels, but are forbidden in vector or tensor channels.

It is plausible ⁵¹, that the same instanton mechanism results in large $\bar{s}s \rightarrow \bar{u}u + \bar{d}d$ transition in $\langle p | j_{\mu 5}^s | p \rangle$, but not in $\langle p | \theta_{\mu\nu}^s | p \rangle$. If it is indeed the case, then the two-component form of s -quark distribution is expected ⁵¹

$$\begin{aligned} s_+(x) + s_-(x) &= (A_1/x)(1-x)^{\beta_s} + (A_2/x)(1-x)^{p'} , \\ s_+(x) - s_-(x) &= B(1-x)^p , \end{aligned} \quad (67)$$

where $\beta_s \approx 5$ and $p' \approx p \approx 10 - 12$. It is easy to see that for distributions (67) the inequality (23) is weakened to $|\Delta s| \lesssim 8V_{2s}$ and (64), (65) are not in contradiction.

In the last year there was an impressive progress in experiment in the problem in view. SMC presented ⁵³ the results of new measurements of $g_{1p}(x)$. The region of x , where the measurement were performed was extended to $0.003 < x < 0.7$. The SMC data are in a good agreement with previous EMC and SLAC data in the domain of x common for all experiments. However, at $x < 0.015$ the SMC points lay higher, than the EMC extrapolation. The SMC data are presented in Fig.2. SMC found for Γ_p at $\bar{Q}^2 = 10 \text{ GeV}^2$

$$\Gamma_p = 0.136 \pm 0.011 \pm 0.011 . \quad (68)$$

In the combined analysis of all data – SLAC, EMC and SMC, it was obtained ⁵³

$$\Gamma_p = 0.142 \pm 0.008 \pm 0.011 . \quad (68')$$

Fig. 2

The SMC data ⁵³ in comparison with the theory. The solid circles (right-hand axes) show $xg_{1p}(x)$, the open boxes (left-hand axes) show $\int_{x_m}^1 g_{1p}(x)dx$. Only statistical errors are shown. The solid square shows the SMC result for Γ_p with statistical and systematic error combined in quadrature. The solid and dashed lines are theoretical predictions ⁴ and ¹¹ correspondingly (see text). The Ellis–Jaffe prediction (63) is marked by the cross.

If gluonic contribution to Γ_p is neglected, then it follows:
from (68):

$$\Sigma = 0.26 \pm 0.14, \quad \Delta s = -0.12 \pm 0.05, \quad (69)$$

from (68')

$$\Sigma = 0.31 \pm 0.13, \quad \Delta s = -0.10 \pm 0.05. \quad (70)$$

Quite recently the E143 group at SLAC ⁵⁴ presented the preliminary result of $g_{1p}(x)$ measurements at $0.029 < x < 0.8$ and $\bar{Q}^2 = 3 \text{ GeV}^2$. E143 found

$$\Gamma_p = 0.133 \pm 0.004 \pm 0.012, \quad \bar{Q}^2 = 3 \text{ GeV}^2. \quad (71)$$

(In obtaining (71) E143 group introduced a correction for $x < 0.029$ region contribution by extrapolation of their data. This extrapolation curve however, lays below the direct SMC data points.) E143 data (71) result in

$$\Sigma = 0.32 \pm 0.12, \quad \Delta s = -0.10 \pm 0.04. \quad (72)$$

The neutron structure function $g_{1n}(x)$ was measured in two experiments: by SMC at CERN in polarized muon scattering on polarized deuterium ⁵⁵ and by E142 group at SLAC in polarized electron scattering on polarized ^3He ⁵⁶. The mean value of \bar{Q}^2 in the SMC experiment was $\bar{Q}^2 = 4.6 \text{ GeV}^2$, in E142 experiment $\bar{Q}^2 = 2 \text{ GeV}^2$. After correction for D -wave admixture in the deuteron, SMC found

$$\Gamma_p + \Gamma_n = 0.050 \pm 0.044 \pm 0.033, \quad \bar{Q}^2 = 4.6 \text{ GeV}^2. \quad (73)$$

In order to determine Γ_n from (73) and Γ_p measurements the data must be taken at one common value $Q^2 = Q_0^2$. We will do such recalculation, in the same way as in ⁵⁷ – accounting for perturbative corrections given in (40) and twist 4 correction (56) and neglecting Δg contribution. As Q_0^2 we choose $Q_0^2 = 10.7 \text{ GeV}^2$. We get

$$\frac{(\Gamma_p + \Gamma_n)|_{Q_0^2}}{(\Gamma_p + \Gamma_n)_{4.6 \text{ GeV}^2}} = 1.05 \quad (74)$$

and the corresponding to (73) value at Q_0^2 is

$$\Gamma_p + \Gamma_n = 0.053 \pm 0.046 \pm 0.035, \quad \bar{Q}_0^2 = 10.7 \text{ GeV}^2. \quad (75)$$

If the result of combined analysis (68') is taken for Γ_p then

$$\Gamma_n = -0.089 \pm 0.04 \pm 0.04 \quad (76)$$

and

$$\Gamma_p - \Gamma_n = 0.231 \pm 0.045 \pm 0.045, \quad Q^2 = Q_0^2. \quad (77)$$

This experimental number may be compared with the Bjorken sum rule, calculated with account of perturbative QCD corrections up to α_s^3 ¹⁸

$$(\Gamma_p - \Gamma_n)_{theor} = 0.186 \pm 0.003, \quad Q^2 = Q_0^2. \quad (78)$$

The two numbers agree within the experimental errors, which, unfortunately, are rather large.

The E142 experiment has the advantage that the spins of two protons in ^3He are compensated and the experiment (up to small correction) gives directly $g_{1n}(x)$. The experiment was done at $0.03 < x < 0.6$. Performing the extrapolation in the regions of small and large x the E142 group obtained

$$\Gamma_{1n} = -0.022 \pm 0.011, \quad \bar{Q}^2 = 2 \text{ GeV}^2 \quad (79)$$

Ellis and Karliner⁵⁷, using E142 data, but different extrapolation obtained instead $\Gamma_{1n} = -0.028$. The value Γ_{1n} close to the latter was obtained by SMC⁵⁸, when they use their data for extrapolation. Transferring the E142 group data to the common value $Q^2 = Q_0^2$, we have from (79)

$$\Gamma_n(Q_0^2) = -0.024 \pm 0.012 \text{ } (-0.030). \quad (80)$$

(In the parenthesis the value, following from analysis^{57,58} is given.) From (68') and (80) it follows for the Bjorken sum rule

$$\Gamma_p - \Gamma_n = 0.166 \pm 0.021 \text{ } (0.172), \quad Q^2 = Q_0^2, \quad (81)$$

in agreement with theoretical value (78). The values of proton spin projection carried by quarks can be also determined from E142 data (79). The results are the following

$$\begin{aligned} \Delta u &= 0.88 \pm 0.05, \Delta d = -0.37 \pm 0.05, \Sigma = 0.47 \pm 0.12 \\ \Delta s &= -0.04 \pm 0.05. \end{aligned} \quad (82)$$

(The difference between (79) and the value given by^{57,58} is included in the error.) The results of all experiments, represented in terms of the part of proton spin projection, carried by u, d and s -quarks are compatible, although some spread in the results is also seen. (Particularly, the difference between the EMC and E142 data.)

Different experiments were done at different \bar{Q}^2 and, even more, each bin in x corresponds to each own Q^2 . The problem arises, if the account of Q^2 dependence could change the results. Experimentally, no Q^2 dependence in the asymmetry A (7) was observed, but the accuracy is not good enough — not better than 10% at high Q^2 and 20% at low Q^2 . In ref.59 by solving the evolution equations the problem was studied, how the fact, that each bin in x is measured at each own Q^2 , can affect the final result. It was found that this effect is small, less than the errors in the present experiments.

Till now when examining the experimental data the higher twist corrections were disregarded, or accounted basing on the calculations ⁴¹, where they are very small, much less than experimental errors. However, as was explained in Sec.5, the results of ⁴¹ are not convincing. Now I will discuss another way of determination of $\Gamma_{p,n}(Q^2)$ Q^2 dependence, based on the connection of $\Gamma_{p,n}(Q^2)$ with GDH sum rule.

Following ^{3,6}, introduce the functions

$$I_{p,n}(Q^2) = \int_{Q^2/2}^{\infty} \frac{d\nu}{\nu} G_{1,p,n}(\nu, Q^2) . \quad (83)$$

Using (2), it is easy to demonstrate, that at large Q^2

$$I_{p,n}(Q^2) \approx \frac{2m^2}{Q^2} \Gamma_{p,n}(Q^2) . \quad (84)$$

At $Q^2 = 0$, according to GDH sum rule, we have

$$I_{p,n}(0) = -\frac{1}{4} \kappa_{p,n}^2 . \quad (85)$$

Fig. 3

The connection of GDH sum rule with sum rules at high Q^2 - the qualitative dependence of $I_p(Q^2)$, $I_n(Q^2)$ and $I_p(Q^2) - I_n(Q^2)$ on Q^2 .

The Q^2 dependence of $I_{p,n}(Q^2)$, and the difference $I_p(Q^2) - I_n(Q^2)$ is plotted

qualitatively in Fig. 3. The case of I_p is especially interesting. At large Q^2 $I_p(Q^2)$ is positive, but at $Q^2 = 0$ it is negative and large, $I_p(0) = -0.72$. This means that in the region $Q^2 \lesssim 1 \text{ GeV}^2$, $\Gamma_p(Q^2)$ varies strongly and changes the sign. Such a behavior cannot be achieved by smooth extrapolation of perturbative QCD corrections and is an indication of large nonperturbative effects.

In ref.6 the vector dominance based model was suggested, which interpolates $I_{p,n}(Q^2)$ at intermediate Q^2 , has the correct asymptotics (84) at large Q^2 and satisfies the GDH sum rule (85) at $Q^2 = 0$. In constructing such model it must be taken into account, that the contribution of baryonic resonances are important in $I_{p,n}(Q^2)$ at low Q^2 ^{60,61}. Therefore, the model for $I_{p,n}(Q^2)$ has the form

$$I_{p,n}(Q^2) = I_{p,n}^{res}(Q^2) + I'_{p,n}(Q^2) , \quad (86)$$

where $I_{p,n}^{res}(Q^2)$ is the contribution of baryonic resonances, known from the analysis of pion electroproduction experiments (up to mass $W = 1.8 \text{ GeV}$) ⁶². $I'_{p,n}(Q^2)$ is defined by

$$I'_{p,n}(Q^2) = 2m^2 \Gamma_{p,n}^{as} \left[\frac{1}{Q^2 + \mu^2} - \frac{c_{p,n} \mu^2}{(Q^2 + \mu^2)^2} \right] , \quad (87)$$

$$c_{p,n} = 1 + \frac{1}{2} \frac{\mu^2}{m^2} \frac{1}{\Gamma_{p,n}^{as}} \left[\frac{1}{4} \kappa_{p,n}^2 + I_{p,n}^{res}(0) \right] , \quad (88)$$

where μ^2 is the vector meson mass, $\mu^2 = 0.6 \text{ GeV}^2$. $\Gamma_{p,n}^{as}$ means the value $\Gamma_{p,n}$ at large Q^2 with higher twist terms excluded. For our purposes — to determine the Q^2 power corrections we may consider $\Gamma_{p,n}^{as}$ as $\Gamma_{p,n}$ at $Q^2 \sim 10 \text{ GeV}^2$. The second term in (87) in the framework of VDM corresponds to the case when in forward virtual γ Compton scattering amplitude both γ interact with the nucleon through vector mesons. The first term in (87) corresponds to the case, when one γ interacts through vector meson and the other one directly. It is easy to see that in the model the asymptotic behavior of $I_{p,n}(Q^2)$ and GDH sum rule is fulfilled. From the experimental data on pion photoproduction, the known values of $\kappa_{p,n}$ and experimental data of $\Gamma_{p,n}^{as}$ it was found ⁶¹

$$c_p = 0.43 \pm 0.10 , \quad c_n = 0.0 \begin{matrix} +0.3 \\ -1.2 \end{matrix} \quad (89)$$

The large error in c_n results from large uncertainty in Γ_n in E142 experiment.

The resonance contribution is absent (or small) in the region of Q^2 and x , where the existing experiments were performed. Then the power corrections came from $I'_{p,n}$ in (86) and can be calculated using (87) - (89). The results are

$Q^2(\text{GeV}^2)$	2	3	4.6	10.5
$\Gamma_p^{as} / \Gamma_p^{exper}$	1.44	1.29	1.19	1.08
$\Gamma_n^{as} / \Gamma_n^{exper}$	1.30	1.20	1.13	1.06

(90)

The results for $\Gamma_{p,n}$, Σ and Δs arising after introduction of power corrections (90) are given in the Table I. (The SMC Γ_n data are not included, because of large uncertainty in Σ and Δs).

Table I

	EMC(p)	SMC(p)	E143(p)	E142(n)
$\Gamma_{p,n}$	0.137 ± 0.018	0.147 ± 0.016	0.172 ± 0.013	-0.029 ± 0.015
Σ	0.28 ± 0.17	0.37 ± 0.14	0.68 ± 0.12	0.46 ± 0.13
Δs	-0.10 ± 0.06	-0.08 ± 0.05	$+0.02 \pm 0.04$	-0.04 ± 0.05

All the data after introduction of power corrections are consistent with $\Sigma \approx 0.4 - 0.5$ and $\Delta s \approx -0.02 - 0.04$ — the values close to naive expectations — and no large gluonic term is needed. The contribution of gluons was disregarded in the calculation of Δu , Δd , Δs and $\Delta \Sigma$, the results of which are presented in (69), (70), (72), (82) as well as in Table I. As was mentioned above, if $\Delta g(10 \text{ GeV}^2) = 0.5$ the account of gluons would increase Σ and Δs by 0.07 and 0.02 correspondingly and twice as large if $\Delta g(10 \text{ GeV}^2) = 1$, what, may be, is also acceptable.

Till now I discussed mostly the integrals of $g_1(x)$ over x , which are known better theoretically than the x -dependence of $g_1(x)$. There are few theoretical models for $g_1(x)$ but, unfortunately, the successful description of the data was achieved by the price of use some experimental information as input. Fig.2 displays the theoretical curve for $g_{1p}(x)$, given by Brodsky ⁴, on the base of matching the Regge behavior at small x with quark counting at large x . An essential point was that the matching was done separately for $q_+(x)$ and $q_-(x)$ ($q = u, d, s$). The values $\Delta u, \Delta d, \Delta s$ from EMC experiment are used as input. The same figure shows also $g_{1p}(x)$ found in the QCD sum rule approach ¹¹. In this calculation no experimental input is used: the only parameter, which enters the calculation is the fundamental QCD parameter - the quark condensate $\alpha_s < 0 \mid \bar{\psi}\psi \mid 0 >^2$. Unfortunately, the domain of x , where the results ¹¹ are reliable (this domain was determined in the calculation) is rather narrow, $0.5 \leq x \leq 0.8$.

9. Suggestions for Future Experiments

1. The experimental study of the sum rule Q^2 dependence. As was discussed in the previous Section, the part of the deviation from the theoretical expectations, as well as the difference between the data, obtained at different Q^2 , may be attributed to high twist effects. Also, the role of Q^2 dependent α_s corrections is not negligible in the problem in view. For this reason the precise measurements of the Q^2 dependence of the sum rules would be very desirable. Such measurements are expected in the near future at SLAC (see ^{63,64}), CERN and DESY (HERMES Collaboration). Especially interesting would be the direct experimental test of the GDH sum rule and the study of Q^2 dependence of $I_{p,n}(Q^2)$ (83) in the domain $0 < Q^2 \lesssim 2 \text{ GeV}^2$. Up to now the GDH sum rule was checked in indirect way, using the data on the pho-

toproduction of baryonic resonances. It was found ^{65,66}, that if the GDH sum rule is decomposed into isoscalar and isovector components, then the isovector–isovector sum rule is well satisfied, isoscalar–isoscalar contribution is small. However, in the isovector–isoscalar sum rule the contribution of high mass resonances is essential, what deteriorates the accuracy of the results. The study of $I_{p,n}(Q^2)$ Q^2 dependence would be important for checking of various models. Particularly, in the model ⁶¹ it is expected, that $I_p(Q^2)$ is crossing zero at $Q^2 \approx 0.5 \text{ GeV}^2$.

2. The search for Δg . The straightforward way to determine $g_+(x) - g_-(x)$ is by measuring two jets in the virtual photon fragmentation region in the polarized $\mu(e)$ –proton deep inelastic scattering ²⁰. Such events arise from $\gamma - g \rightarrow \bar{q}q$ collisions and may be separated from other two jet events by their known k_\perp^2 dependence ²⁰. When the jets are well separated in k_\perp^2 , there are no infrared problems and the cross section is predicted uniquely in terms of $g_+(x) - g_-(x)$.

For discussion of other possibilities see the review ³² and references herein.

3. The measurements of Δs . Besides of Δs determination in future precise polarized deep inelastic $\mu(e)$ –nucleon scattering, Δs can also be measured in elastic νp scattering by separating axial formfactor contribution at zero momentum transfer ⁶⁷. The axial formfactor arises from Z –boson exchange and at $Q^2 = 0$ is proportional to the matrix element

$$\langle p | \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d - \bar{s}\gamma_\mu\gamma_5 s | p \rangle = -2s_\mu m(\Delta u - \Delta d - \Delta s) . \quad (91)$$

Such an experiment gives an additional equation for determination of $\Delta u, \Delta d, \Delta s$. The existing data ⁶⁸ indicate on large Δs

$$\Delta s = -0.15 \pm 0.08 . \quad (92)$$

The determination of axial formfactor $G_A(0)$ from the data must be done with some care. The experiment is performed at $Q^2 \neq 0$ and the extrapolation to $Q^2 = 0$ proceeds, assuming the dipole formula for the formfactor,

$$G_A(Q^2) = \frac{G_A(0)}{[1 + Q^2/M_A^2]^2} . \quad (93)$$

However, due to chirality conservation, (93) cannot be correct in the whole domain of Q^2 . This follows from the requirement that at large Q^2 the axial formfactor must coincide with vector one ⁶⁹. Therefore a deviation from the dipole fit (93) is expected at intermediate Q^2 . This expectation is in accord with the discussed in Sec.8 nonperturbative effects (instantons) in flavour singlet axial formfactor. Surely, the observation of the deviation from the dipole fit in singlet axial would be extremingly important for our understanding of nonperturbative QCD.

The measurements of s –quark distribution in nonpolarized nucleon would be also helpful in understanding the origin of the strange sea. The s –quark distribution

obtained from the data ⁷⁰ on charm production in νN scattering shows a more steep $1 - x$ dependence

$$xs(x) \sim (1 - x)^\beta, \quad \beta \approx 6.5,$$

then the quark counting rule expectation $\beta \approx 5$. This may be an indication in favour of nonperturbative (instanton) component (67) in s -quark distribution, but more precise data are needed.

If Δs is large and negative, then a surplus of jets with leading strange particles can be expected in the direction opposite to the proton spin. This also could be tested.

4. The check of the OZI rule in axial mesons. A connected problem is the determination if OZI rule holds in the nonet of axial mesons. If Δs is large and nonperturbative effects are important in singlet axial channel, then it is expected ⁴⁸ a noticeable OZI rule violation in the nonet of axial 1^{++} mesons. There are indications, that it is indeed, what happened ⁷¹.

10. The Structure Function $g_2(x)$

Since there are very good reviews ^{72,32} on this subject and almost nothing new happened after their appearance, I will indicate here the basic points only. The structure function $g_2(x)$ can be measured in deep inelastic $\mu(e)$ -nucleon scattering, when $\mu(e)$ is polarized longitudinally and nucleon transverse to the beam direction. In this case the contributions of g_1 and g_2 to spin asymmetry (measured, e.g. by changing the direction of nucleon spin to the opposite) are of the same order, but suppressed by the factor $\sim 1/Q$ in comparison with the g_1 contribution in longitudinally polarized scattering. In the cases of other directions of nucleon spin g_2 contribution is smaller, than g_1 by a factor $\sim 1/Q$. This fact demonstrates that g_2 may be considered as a twist-3 structure function.

For free particle $g_2 \equiv 0$. This is an indication, that g_2 cannot be described in terms of parton distributions, because in the parton model we consider deep inelastic scattering, as proceeding on free partons. The other argument in the same direction can also be presented. Let us suppose, following Feynman ⁷³ that the transversely polarized nucleon in the infinite momentum frame is described by parton densities $t_j^\pm(x)$ with polarizations ± 1 upon nucleon polarization direction. Then, it can be shown ³, that

$$g_1(x) + g_2(x) = \sum e_j^2 \left(\frac{\mu_j}{m} + \frac{\overline{p_{\perp j}^2}}{mQ} \right) \frac{1}{2x} [t_j^+(x) - t_j^-(x)] . \quad (94)$$

Here μ_j, e_j and $\overline{p_{\perp j}^2}$ are the mass, charge and mean square of the transverse momentum of parton j . At large Q , since μ_j are the current masses, the r.h.s. of (94) is small. But the integral over x of the l.h.s. is not small, according to sum rules, so a contradiction arises. The quark-gluon interaction and transverse momentum quark

distribution in nucleon are of importance in $g_2(x)$ determination. Both of them are beyond the parton model. For this reason our theoretical knowledge of $g_2(x)$ is very scarce now.

In OPE on the light cone $g_2(x)$ can be decomposed into two sets of operators⁷⁴. The first is twist-2 operators, the same, which appear in $g_1(x)$ decomposition, the second is twist-3 operators. The first set contribution can be expressed through $g_1(x)$ and the relation can be obtained⁷⁴

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2) , \quad g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dz}{z} g_1(z, Q^2) , \quad (95)$$

where g_2^{WW} and \bar{g}_2 correspond to twist-2 and 3. There is no reason to believe, that the most interesting piece — $g_2(x, Q^2)$, determined by quark-gluon interaction, is much less than $g_2^{WW}(x, Q^2)$. Therefore, it is unclear, if decomposition (95) is helpful.

It is very important to check experimentally the Burkhardt-Cottingham (BC) sum rule (30). At such investigation it would be interesting to study Q^2 dependence of the sum rule: as was argued in Sec.4 one may expect that BC sum rule is valid only in low twist order and at $Q^2 \sim 1 - 2 \text{ GeV}^2$ the violation of BC sum rule could be seen.

11. Chirality Violating Structure Function $h_1(x)$

As is well known, all structure functions of the twist two - $F_1(x)$, $F_2(x)$, $g_1(x)$ which are measured in the deep-inelastic lepton-nucleon scattering conserve chirality. Ralston and Soper⁷⁵ first demonstrated that besides these structure functions, there exists the twist-two chirality violating nucleon structure function $h_1(x)$. This structure function does not manifest itself in the deep inelastic lepton-hadron scattering, but can be measured in the Drell-Yan process with both beam and target transversally polarized. The reason of this circumstance is the following. The cross section of the deep inelastic electron(muon)-hadron scattering is proportional to the imaginary part of the forward virtual photon-hadron scattering amplitude. At high photon virtuality the quark Compton amplitude dominates, where the photon is absorbed and emitted by the same quark (Fig.4a) and the conservation of chirality is evident. The cross section of the Drell-Yan process can be represented as an imaginary part of the diagram, Fig.4b. Here virtual photons interact with different quarks and it is possible, as is shown in Fig.4b, that chirality violating amplitude in Drell-Yan processes is not suppressed at high Q^2 in comparison with chirality conserving ones, and consequently, corresponds to twist two. However, this amplitude, corresponding to target spin flip, has no parton interpretation in terms of quark distributions in the helicity basis and, as was shown by Jaffe and Ji⁷⁶ can be only represented as an element of the quark-quark density matrix in this basis.

Fig. 4

a) Deep inelastic lepton-hadron scattering, the quark chiralities are conserved. Solid lines are quarks, wavy lines are virtual photons, $R(L)$ denote right (left) chirality of quarks; b) Drell-Yan process with chirality of quarks flipped.

However, $h_1(x)$ can be interpreted ⁷⁶ as a difference of quark densities with the eigenvalues $+1/2$ and $-1/2$ of the transverse Pauli-Lubanski spin operator $\hat{s}_\perp \gamma_5$ in the transversely polarized proton. In this basis $h_1(x)$ can be described in terms of standard parton language.

Until now there are no experimental data on the chirality violating nucleon structure function $h_1(x)$. Besides ^{75,76} the theoretical study of this structure function have been also performed by Artru and Mekhfi ⁷⁷. The role of $h_1(x)$ in factorization of a general hard process with polarized particles was investigated by Collins ⁷⁸. The first attempt to calculate $h_1(x)$ was carried out by Jaffe and Ji ⁷⁶ by means of the bag model.

The proton structure function $h_1(x)$ can be defined in the light cone formalism as follows ⁷⁶

$$i \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p, s | \bar{\psi}(0) \sigma_{\mu\nu} \gamma_5 \psi(\lambda n) | p, s \rangle = 2[h_1(x, Q^2)(s_{\perp\mu} p_\nu - s_{\perp\nu} p_\mu) + h_L(x, Q^2)m^2(p_\mu n_\nu - p_\nu n_\mu)(sn) + h_3(x, Q^2)m^2(s_{\perp\mu} n_\nu - s_{\perp\nu} n_\mu)] . \quad (96)$$

Here n is a light cone vector of dimension $(\text{mass})^{-1}$, $n^2 = 0$, $n^+ = 0$, $pn = 1$, p and s are the proton momentum and spin vectors, $p^2 = m^2$, $s^2 = -1$, $ps = 0$ and $s = (sn)p + (sp)n + s_\perp$, $h_L(x, Q^2)$ and $h_3(x, Q^2)$ are twist-3 and 4 structure functions. For comparison in the same light cone notation the standard structure function $F_1(x, Q^2)$ is given by

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p, s | \bar{\psi}(0) \gamma_\mu \psi(\lambda n) | p, s \rangle = 4[F_1(x, Q^2)p_\mu + M^2 f_4(x, Q^2)n_\mu] \quad (97)$$

(Eq. (96), (97) are written for one flavour).

Basing on the definition (96) an inequality was proved in ⁷⁶

$$q(x) \geq h_1^q(x) , \quad (98)$$

which holds for each flavour $q = u, d, s$. (Here $h_1^q(x)$ is the flavour q contribution to $h_1(x)$.) It was also suggested ⁷⁶, that $|h_1^q(x)| > |g_1^q(x)|$.

$h_1(x)$ can be also represented through a T -product of currents ⁷⁹

$$T_\mu(p, q, s) = i \int d^4x e^{iqx} \langle p, s | (1/2)T\{j_{\mu 5}(x), j(0) + j(x), j_{\mu 5}(0)\} | p, s \rangle , \quad (99)$$

where $j_{\mu 5}(x)$ and $j(x)$ are axial and scalar currents.

The general form of $T_\mu(p, q, s)$ is

$$T_\mu(p, q, s) = \left(s_\mu - \frac{qs}{q^2} q_\mu \right) \tilde{h}_1(x, Q^2) + \left(p_\mu - \frac{\nu q_\mu}{q^2} \right) (qs) l_1(x, Q^2) + \varepsilon_{\mu\nu\lambda\sigma} p_\nu q_\lambda s_\sigma (qs) l_2(x, q^2) \quad (100)$$

(only spin-dependent terms are retained). It can be proved ⁷⁹, that

$$h_1(x, Q^2) = -\frac{1}{\pi} Im \tilde{h}_1(x, Q^2) . \quad (101)$$

As is clear from (96) or (99) $h_1(x)$ indeed violates chirality. It is worth mentioning that instead of axial and scalar currents in (99) the combination of vector and pseudoscalar currents $j_\mu(x)j_s(0) - j_s(x)j_\mu(0)$ can also be used (notice the minus sign in the crossing term).

The structure function $h_1(x)$ can be measured in the production of $\mu^+\mu^-$ or e^+e^- pairs in pp (or $p\bar{p}$) collisions, when both protons (or p and \bar{p}) are polarized transversally to the beam direction. Introduce the coordinate system, where the beam direction is along z , the proton spin direction—along x and the momentum of the lepton pair is characterized by polar and azimuthal angles θ and φ (in the lepton pair c.m.s.). Then the asymmetry is given by ^{75,80}

$$A(x, y) = \frac{\sigma_+(x, y) - \sigma_-(x, y)}{\sigma_+(x, y) + \sigma_-(x, y)} = \frac{\sin \theta \cos 2\varphi}{(1 + \cos^2 \theta)} \frac{\sum_a e_a^2 h_1^a(x) h_1^{\bar{a}}(y)}{\sum_a e_a^2 q^a(x) q_1^{\bar{a}}(y)} \quad (102)$$

Here $\sigma_+(x, y)$ and $\sigma_-(x, y)$ are the cross sections of lepton pair production, where the spin of one of protons is along or opposite to spin of the other (or \bar{p}).

In (102) x and y are the part of the momenta, carried by quarks and antiquarks, annihilating into virtual photon with momentum q

$$\begin{aligned} q &= xp_1 + yp_2 + q_\perp , \\ M^2 &= q^2 = xs - \mathbf{q}_\perp^2 , \end{aligned} \quad (103)$$

where M is the mass of leptonic pair. h_1^a and $h_1^{\bar{a}}$ are the structure functions h_1 for quarks and antiquarks in the target and projectile correspondingly, q^a and $q^{\bar{a}}$ are the quark and antiquark distributions. In $p\bar{p}$ collisions $h_1^a = h_1^{\bar{a}}$, $q^a = q^{\bar{a}}$.

Since $h_1(x)$ violates chirality one may expect that in QCD it can be expressed in terms of chirality violating fundamental parameters of the theory, the simplest of which (of lowest dimension) is the quark condensate. Basing on this idea $h_1(x)$ calculation was performed in ⁷⁹. The general method ^{81,82} of structure function calculations in the QCD sum rule approach was used. In this method the structure function at intermediate x can be found. The main difference in comparison with ⁸²

is that in the case of $h_1(x)$ determination the chirality violating structure is studied. As a result $h_1(x)$ was found to be proportional to the quark condensate $\langle 0 | \bar{u}u | 0 \rangle$ with the correction term in OPE proportional to the mixed quark–gluon condensate $g \langle 0 | \bar{u}\sigma_{\mu\nu}G_{\mu\nu}^n\lambda^nu | 0 \rangle$. It was obtained that for proton $h_1^u(x) \gg h_1^d(x)$, and as a consequence $h_1(x) \approx (4/9)h_1^u(x)$. The calculations in ⁷⁹ are valid at $0.3 < x < 0.55$. The extrapolation in the region of small x can be performed using Regge behavior $h_1(x) \sim x^{-\alpha_{a1}}$, the extrapolation in the region of large x , using the inequality (98). The final result of calculation of $h_1^u(x)$ with the extrapolations is shown in Fig.5.

Fig. 5

The u -quark contribution to the proton structure function $h_1(x)$ based on QCD sum rule calculation ⁷⁹ at intermediate x , $0.3 < x < 0.55$. At $x < 0.3$ an extrapolation according to Regge behavior was performed, at $x > 0.55$ (dashed line) the saturation of inequality (98) was assumed.

It is expected that the accuracy of $h_1^u(x)$ determination is about 30% at $x \approx 0.4$ and about 50% at $x = 0.6$. The inequality $h_1^u(x) > g_1^u(x)$ suggested in ⁷⁶ was confirmed. Numerically $h_1(x)$ is rather large, what gives a good chance for its experimental study.

It is difficult to produce intense high energy polarized antiproton beam ($\bar{p}p$ collisions are preferable for the study of h_1 structure function, since the same h_1^a enters (102) for target and projectile). For this reason there are suggestions of experiments, where only target proton is polarized, but also the polarization of the final particle is measured ^{83–86}

12. C o n c l u s i o n s

In the last year the situation, connected with the so called “proton spin crises” became less dramatic, than it was before, after the appearance of the first EMC data. Although there is no contradiction between the new and the old data, the world averages moved towards theoretical expectations. The simplest theoretical prediction—the Ellis–Jaffe sum rule value moved slightly towards experimental number after appearance of new data on hyperon β -decay. As a result, the gap between the experimental value of Γ_p and Ellis–Jaffe sum prediction shrinked. The new neutron data are more or less in agreement with theoretical expectations and there is no contradiction with the Bjorken sum rule.

On the other side the theoretical understanding of the nucleon spin content increased essentially: we understand now that Ellis–Jaffe sum rule is not the last word in the problem, we have more knowledge about the role of gluons and strange quarks. Since the gap between the theoretical expectation given by Ellis–Jaffe sum rule and world averaged experimental value became narrower, a possibility appeared to share this difference between the sources, nonaccounted in Ellis–Jaffe sum rule (strange sea, gluons, higher twist terms) in more or less conservative way. For example, such a scenario is not in contradiction with experimental data: at $Q^2 = 10 \text{ GeV}^2$ $\Delta g \approx 0.5 - 1.0$, $\Delta s \approx -0.02 - 0.05$, $\Sigma \approx 0.4 - 0.5$ and higher twist corrections comprising $\sim 4\%$ (the latter two times less than in Table 1).

Evidently, these numbers would surprise nobody and would not require any revolutionary changes in our understanding of nucleon structure. The problem is: if this or a similar scenario is realized in the nature. Future, more precised experiments on deep inelastic $\mu(e)$ nucleon scattering will help us to obtain the answer this question. I emphasize, that these experiments, even very precised, are not enough for the final solution of the problem: an independent experiments, measuring the part of the nucleon spin carried by strange quarks and gluons are needed. A new era in the investigation of the nucleon spin structure will open, when the measurements with the transversally (relative to the beam) polarized nucleon will start. May be new surprises are waiting us here.

I am grateful to J.Lichtenstadt for the discussion of experimental results and for the information about new data and to M.Karliner for helpful discussion of new theoretical developments.

This work was supported in part by International Science Foundation Grant M9H000 and by International Association for the Promotion of Cooperation with Scientists from the Independent States of the Former Soviet Union Grant INTAS–93–283.

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